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Moment equation description of Weibel instability

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A macroscopic description of the linear Weibel instability, based on a closed set of linear moment equations, is presented. The moment equations are derived from the linearized Vlasov equation by taking the appropriate velocity moments of it and the closure is achieved by means of an assumption, which is justified when the temperature anisotropy is strong. The macroscopic description is manifestly more informative of the physical mechanism of the instability than the kinetic description. It is hoped that the researchers will find such a description analytically more convenient to use in solving plasma physics problems where Weibel instability due to strong temperature anisotropy plays a role. [DOI: 10.1063/1.1521716]

Particle distributions in velocity space with an enhanced temperature along some direction are quite common in both laboratory and space plasmas where the degree of collisionality is relatively low. There are a number of ways in which such a temperature anisotropy can develop in magnetically confined as well as in magnetic field-free plasma.[1] Coulomb collisions eventually make the plasma distribution isotropic. However, collective processes (instabilities) due to the temperature anisotropy can be more effective than the binary collisions in moving the plasma toward an isotropic state. These instabilities can be both electrostatic and electromagnetic in nature.[2] Here we shall deal with a particular electromagnetic instability due to temperature anisotropy, known as the Weibel[3] instability, which is excited in collisionless plasma, even in the absence of an external magnetic field. The unstable waves are transverse electromagnetic waves, involve only the electron population, and do not produce density perturbation. The Weibel instability is considered to be one of the mechanisms for magnetic field generation[4,5] in laser produced plasma, with special significance in laser fusion experiments. It may also play an important role in the magnetic reconnection process in the Earth's magnetotail by participating in the dynamics of the electrons in the so-called electron diffusion region.[6]

Weibel's work stimulated a series of further investigations[7–14] of the transverse electromagnetic instability in unmagnetized plasma. These papers dealt with the linear, quasilinear and fully nonlinear theories as well as the computer simulation experiments of the instability. At the same time, several other authors[15–21] investigated the electromagnetic instabilities in magnetized plasma for a wide variety of anisotropic velocity distributions and for different orientations of the propagation vector. Most recently, Califano *et al.*[22,23] have carried out further investigations of Weibel-type instability, where the role of temperature anisotropy is taken by two counterstreaming electron populations. All of the previous analyses, including that of Weibel, have been based on the Vlasov–Maxwell formalism.

In the Vlasov–Maxwell formalism, the linear dispersion relation for Weibel instability in unmagnetized plasma is derived by solving the linearized, nonrelativistic Vlasov equation,

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\tilde{f}(\mathbf{r},\mathbf{v},t)-\frac{e}{m}\left(\tilde{\mathbf{E}}+\frac{1}{c}\mathbf{v}\times\tilde{\mathbf{B}}\right)\cdot\frac{\partial}{\partial\mathbf{v}}f_0(\mathbf{v})=0, \tag{1}$$

and the two Maxwell's equations,

$$\nabla\times\tilde{\mathbf{E}}=-\frac{1}{c}\frac{\partial\tilde{\mathbf{B}}}{\partial t}, \tag{2}$$

$$\nabla\times\tilde{\mathbf{B}}=-\frac{4\pi e}{c}\int d\mathbf{v}\mathbf{v}\tilde{f}(\mathbf{r},\mathbf{v},t)+\frac{1}{c}\frac{\partial\tilde{\mathbf{E}}}{\partial t}, \tag{3}$$

self-consistently. Here $f_0(\mathbf{v})$ is the unperturbed electron velocity distribution, $\tilde{f}(\mathbf{r},\mathbf{v},t)$ is the perturbation of the distribution, and $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ are the perturbed electric and magnetic fields, respectively. Considering

$$f_0(\mathbf{v})=\frac{n_0}{T_\parallel^{1/2}T_\perp}\left(\frac{m}{2\pi}\right)^{3/2}\exp\left[-\frac{m}{2T_\perp}(v_x^2+v_y^2)-\frac{m}{2T_\parallel}v_z^2\right], \tag{4}$$

and space–time dependence of the perturbations of the form $\sim\exp[i(kz-\omega t)]$, the linear dispersion relation of the transverse waves ($\mathbf{k}\cdot\tilde{\mathbf{E}}=0$) is then obtained as

$$\omega^2-c^2k^2-\omega_{pe}^2\left[1+\frac{T_\perp}{T_\parallel}W\left(\frac{\omega}{kV_{T_\parallel}}\right)\right]=0. \tag{5}$$

Here $\omega_{pe}=(4\pi e^2n_0/m)^{1/2}$ is the electron plasma frequency, $V_{T_\parallel}=(2T_\parallel/m)^{1/2}$ is the electron thermal speed associated with $T_\parallel$, and $W(\xi)=-1-\xi Z(\xi)$, where $Z(\xi)$ is the plasma dispersion function. Weibel[3] demonstrated instability by considering the $|\omega/(kV_{T_\parallel})|\gg1$ limit of Eq. (5). A more systematic analysis of Eq. (5) can be found in Ref. 14. The results are summarized below.

For small deviations from isotropy, $(T_\perp-T_\parallel)/T_\parallel\ll1$, instability is found for a low frequency mode such that

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$|\omega/(kV_{T_\parallel})|\ll 1$. With the substitution of $W(\xi)\cong -1-i\pi^{1/2}\xi$ for $|\xi|\ll 1$, and the use of $\omega\ll ck$, Eq. (5) becomes

$$1+\frac{\omega_{pe}^2}{c^2k^2}\left[1-\frac{T_\perp}{T_\parallel}\left(1+i\sqrt{\pi}\frac{\omega}{kV_{T_\parallel}}\right)\right]=0, \tag{6}$$

where $k>0$ has been assumed. The solution for ω is then given by[14]

$$\omega=i\frac{kV_{T_\parallel}}{\sqrt{\pi}}\frac{T_\parallel}{T_\perp}\left(\frac{T_\perp}{T_\parallel}-1-\frac{c^2k^2}{\omega_{pe}^2}\right). \tag{7}$$

It shows that purely growing waves are excited when $T_\perp > T_\parallel$ and for the values of k in the range $0<k<k_m$, where $k_m^2=(\omega_{pe}^2/c^2)(T_\perp/T_\parallel-1)$.

For strong anisotropy, $(T_\perp-T_\parallel)/T_\parallel\gg 1$, the approximation $|\omega/(kV_{T_\parallel})|\ll 1$ with ω given by Eq. (7) tends to break down, except when $k\approx k_m$. For $k\ll k_m$, the instability is found for frequency ω such that $|\omega/(kV_{T_\parallel})|\gg 1$. In this regime, $W(\xi)\cong 1/(2\xi^2)$, and so Eq. (5) becomes

$$\omega^2-c^2k^2-\omega_{pe}^2\left(1+\frac{k^2T_\perp}{m\omega^2}\right)=0. \tag{8}$$

For $\omega\ll ck$, this again yields a purely growing solution given by[3]

$$\omega=ik\left(\frac{T_\perp}{m}\right)^{1/2}\left(\frac{\omega_{pe}^2}{c^2k^2+\omega_{pe}^2}\right)^{1/2}. \tag{9}$$

It may be verified that the restriction $|\omega/(kV_{T_\parallel})|\gg 1$, with ω given by Eq. (9), is satisfied only when $T_\perp\gg T_\parallel$.

In this Brief Communication, we present a macroscopic description of the linear Weibel instability for the strong anisotropy $(T_\perp\gg T_\parallel)$ case. The description is based upon a closed set of linear moment equations that are derived from the linearized, nonrelativistic Vlasov equation by taking the appropriate velocity moments of it. The derivation requires integration by parts and use of the symmetry properties of f_0 in velocity space. The closure is achieved by means of an assumption, which is justified when $|\omega/(kV_{T_\parallel})|\gg 1$, i.e., when the temperature anisotropy is strong. A similar macroscopic description for the weak anisotropy case is not possible since the instability relies on a wave–particle resonance process.

We consider transverse waves propagating in the z-direction, so that $\tilde{E}_z=\tilde{B}_z=0$. By taking the successive velocity moments of Eq. (1), we find the following interconnected chain of moment equations:

$$\frac{\partial}{\partial t}\tilde{u}_x=-\frac{1}{mn_0}\frac{\partial}{\partial z}\tilde{P}_{xz}-\frac{e}{m}\tilde{E}_x, \tag{10}$$

$$\frac{\partial}{\partial t}\tilde{P}_{xz}+\frac{\partial}{\partial z}\tilde{Q}_{xzz}=\frac{e}{mc}[(\tilde{\mathbf{B}}\times\mathbf{P}_0)_{xz}+(\tilde{\mathbf{B}}\times\mathbf{P}_0)_{zx}], \tag{11}$$

$$\frac{\partial}{\partial t}\tilde{Q}_{xzz}+m\frac{\partial}{\partial z}\int d\mathbf{v}\mathrm{v}_x\mathrm{v}_z^3\tilde{f}(\mathbf{r},\mathbf{v},t)=-\frac{e}{m}P_{0zz}\tilde{E}_x, \tag{12}$$

and so on. These are the linearized, one-dimensional versions of the moment equations[24] that can be obtained from the exact Vlasov equation. Here $\tilde{u}_x$ is the x-component of the perturbed electron mean velocity $\tilde{\mathbf{u}}$, $\tilde{P}_{xz}$ is the xz-element of the perturbed stress tensor $\tilde{\mathbf{P}}$, $\tilde{Q}_{xzz}$ is the xzz-element of the perturbed heat flow tensor $\tilde{\mathbf{Q}}$, and their standard definitions are

$$\tilde{u}_x(\mathbf{r},t)=\frac{1}{n_0}\int d\mathbf{v}\mathrm{v}_x\tilde{f}(\mathbf{r},\mathbf{v},t), \tag{13}$$

$$\tilde{P}_{xz}(\mathbf{r},t)=m\int d\mathbf{v}\mathrm{v}_x\mathrm{v}_z\tilde{f}(\mathbf{r},\mathbf{v},t), \tag{14}$$

$$\tilde{Q}_{xzz}(\mathbf{r},t)=m\int d\mathbf{v}\mathrm{v}_x\mathrm{v}_z^2\tilde{f}(\mathbf{r},\mathbf{v},t). \tag{15}$$

The unperturbed stress tensor $\mathbf{P}_0$, corresponding to the f_0 given by Eq. (4), is diagonal and its elements are $P_{0xx}=P_{0yy}=n_0T_\perp$ and $P_{0zz}=n_0T_\parallel$, the other elements being zero. The notations on the right hand side of Eq. (11) mean xz- and zx-elements of the tensors resulting from the cross products. The moment equations are the exact consequences of taking the appropriate velocity moments of Eq. (1), using the symmetry properties of f_0 in velocity space. However, they do not form a closed set, since each equation contains a term of the higher-order moment. We truncate the chain of equations by assuming $m(\partial/\partial z)\int d\mathbf{v}\mathrm{v}_x\mathrm{v}_z^3\tilde{f}(\mathbf{r},\mathbf{v},t)\ll(\partial\tilde{Q}_{xzz}/\partial t)$, so that Eq. (12) reduces to

$$\frac{\partial}{\partial t}\tilde{Q}_{xzz}\cong-\frac{e}{m}P_{0zz}\tilde{E}_x. \tag{16}$$

Simple order-of-magnitude estimates indicate that the validity of the assumption requires $|\omega/(kV_{T_\parallel})|\gg 1$ for a wave propagating along the z-direction with frequency ω and wave number k. We shall examine this assumption in more details later in the paper.

Equations (10), (11), and (16) together with the two Maxwell's equations, $c(\nabla\times\tilde{\mathbf{E}})=-\partial\tilde{\mathbf{B}}/\partial t$ and $c(\nabla\times\tilde{\mathbf{B}})=-4\pi en_0\tilde{\mathbf{u}}+\partial\tilde{\mathbf{E}}/\partial t$, provide a closed macroscopic description of the linear Weibel instability in the regime corresponding to $|\omega/(kV_{T_\parallel})|\gg 1$. To verify this, we substitute solutions of the form $\tilde{A}(\mathbf{r},t)=\hat{A}(k,\omega)\exp[i(kz-\omega t)]$ into Eqs. (10), (11), and (16) to find

$$\hat{u}_x=\frac{k}{mn_0\omega}\hat{P}_{xz}-\frac{ie}{m\omega}\hat{E}_x, \tag{17}$$

$$\hat{P}_{xz}=\frac{k}{\omega}\hat{Q}_{xzz}+\frac{ien_0}{mc\omega}(T_\parallel-T_\perp)\hat{B}_y, \tag{18}$$

$$\hat{Q}_{xzz}\cong-\frac{ien_0T_\parallel}{m\omega}\hat{E}_x. \tag{19}$$

From Eqs. (18) and (19) we have

$$\hat{P}_{xz}=-\frac{ien_0kT_\parallel}{m\omega^2}\hat{E}_x+\frac{ien_0}{mc\omega}(T_\parallel-T_\perp)\hat{B}_y, \tag{20}$$

and then combining Eq. (20) with Eq. (17) we find

$$\hat{u}_x=-\frac{ie}{m\omega}\left[\left(1+\frac{k^2T_\parallel}{m\omega^2}\right)\hat{E}_x-\frac{k(T_\parallel-T_\perp)}{mc\omega}\hat{B}_y\right]. \tag{21}$$

Finally, combining $\hat{E}_x=(\omega/ck)\hat{B}_y$ and $\hat{B}_y=-(4\pi i/ck)en_0\hat{u}_x+(\omega/ck)\hat{E}_x$, which follow from the Maxwell's equations, with Eq. (21) we obtain the dispersion relation

$$\omega^2-c^2k^2-\omega_{pe}^2\left(1+\frac{k^2T_\perp}{m\omega^2}\right)=0, \tag{22}$$

which is exactly the dispersion relation that is obtained from the Vlasov–Maxwell formalism in the limit $|\omega/(kV_{T_\parallel})|\gg 1$ [see Eq. (8)]. We recall that the condition $|\omega/(kV_{T_\parallel})|\gg 1$ corresponds to the strong temperature anisotropy case.

Let us now return to the assumption by which the term $m(\partial/\partial z)\int d\mathrm{v}\mathrm{v}_x\mathrm{v}_z^3\tilde{f}(\mathbf{r},\mathbf{v},t)$ in Eq. (12) was neglected and thus closure was achieved. If we multiply Eq. (1) by $\mathrm{v}_x\mathrm{v}_z^3$, integrate over velocity space using the symmetry properties of f_0 and neglect the higher-order velocity moment term, we find

$$\frac{\partial}{\partial t}\int d\mathrm{v}\mathrm{v}_x\mathrm{v}_z^3\tilde{f}(\mathbf{r},\mathbf{v},t)\cong\frac{3en_0T_\parallel}{m^3c}(T_\parallel-T_\perp)\tilde{B}_y. \tag{23}$$

So, had we retained the term $m(\partial/\partial z)\int d\mathrm{v}\mathrm{v}_x\mathrm{v}_z^3\tilde{f}(\mathbf{r},\mathbf{v},t)$ in the equation for $\tilde{Q}_{xzz}$ then instead of Eq. (20) we would have obtained

$$\hat{P}_{xz}=-\frac{ien_0kT_\parallel}{m\omega^2}\hat{E}_x+\frac{ien_0}{mc\omega}(T_\parallel-T_\perp)\hat{B}_y+\frac{ien_0}{mc\omega}\left(\frac{3k^2T_\parallel}{m\omega^2}\right)(T_\parallel-T_\perp)\hat{B}_y, \tag{24}$$

where the last term on the right hand side of Eq. (24) is the additional term. It is clear that the additional term may be neglected in comparison with the second term when $|\omega/(kV_{T_\parallel})|\gg 1$. Similarly, it can be shown that the contributions to $\hat{P}_{xz}$ from higher-order velocity moment terms are of higher order in $\varepsilon\equiv|kV_{T_\parallel}/\omega|\ll 1$, and so are even smaller. Hence, the closure of the chain of moment equations by adopting Eq. (16) is indeed justified when $|\omega/(kV_{T_\parallel})|\gg 1$.

It should be pointed out that an equivalent macroscopic description of the instability is obtained if the equations for $\tilde{u}_y$, $\tilde{P}_{yz}$ and $\tilde{Q}_{yzz}$ are considered instead. These equations are readily obtained from Eqs. (10), (11), and (16) with x replaced by y everywhere. In other words, there are two linearly independent eigenstates, represented by $\{\tilde{u}_x,\tilde{P}_{xz},\tilde{Q}_{xzz}\}$ and $\{\tilde{u}_y,\tilde{P}_{yz},\tilde{Q}_{yzz}\}$, respectively, with the same eigenvalue ω_k. This degeneracy, which is a consequence of the symmetry of $f_0(\mathbf{v})$ in velocity space, is removed when an external magnetic field $\mathbf{B}_0$ is applied. The moment equation description presented above can be extended to the magnetized plasma case in a straightforward manner by noting that Eq. (1) will have an additional term represented by $-(e/mc)\times(\mathbf{v}\times\mathbf{B}_0)\cdot(\partial\tilde{f}/\partial\mathbf{v})$, and can be used to study the effects of the external magnetic field on the Weibel instability. For example, if we take $\mathbf{B}_0$ to be along the z-direction, the additional term leads to coupling between the states $\{\tilde{u}_x,\tilde{P}_{xz},\tilde{Q}_{xzz}\}$ and $\{\tilde{u}_y,\tilde{P}_{yz},\tilde{Q}_{yzz}\}$ due to cyclotron motion of electrons. Consequently, the corresponding equations for $\tilde{u}_y$, $\tilde{P}_{yz}$ and $\tilde{Q}_{yzz}$ are needed. Those equations can be derived in a similar manner using the same closure approximation. It is found that, in magnetized plasma, circularly polarized waves are the proper eigenmodes with different dispersion relations for the right and the left circular polarizations. Furthermore, the perturbed electric field associated with the transverse waves propagating only along the z-direction has no electrostatic part. It arises solely from the fluctuating magnetic field.

The macroscopic description, based on the moment equations, is manifestly more informative of the physical mechanism of the Weibel instability than its kinetic description. The proposed moment equations and the subsequent analysis leading to the derivation of the dispersion relation show that the stress tensor term $\tilde{P}_{xz}$ (or $\tilde{P}_{yz}$) in the momentum balance equation plays a crucial role in the excitation of the instability. Without this term, one would recover the familiar stable electromagnetic mode in plasma, which is described by $\omega^2=c^2k^2+\omega_{pe}^2$. The analysis further shows that, in order to accurately determine $\tilde{P}_{xz}$ (or $\tilde{P}_{yz}$), the heat flow term $\tilde{Q}_{xzz}$ (or $\tilde{Q}_{yzz}$) describing perpendicular (to $\mathbf{k}$) flow of the parallel (to $\mathbf{k}$) thermal energy due to the transverse electric field $\tilde{E}_x$ (or $\tilde{E}_y$) must be retained. The physical mechanism represented by the moment equations is that the force experienced by the electrons in the fluctuating field and the associated heat flux cause a momentum flux ($\tilde{\mathbf{P}}$), which affects $\tilde{\mathbf{u}}$ (and hence current density $\tilde{\mathbf{j}}$) in such a way as to increase the field fluctuation. Earlier, Fried[25] offered a similar explanation by means of an approximate treatment, and the role of heat flow was not recognized. The moment equation description presented here seems to provide a more complete and accurate picture of the instability. A kinetic description of linear Weibel instability can become quite cumbersome analytically, if the equilibrium magnetic field is inhomogeneous as, for example, is the case with the Earth's magnetotail, since it involves the calculation of unperturbed particle orbits and the integration of the linearized Vlasov equation along those orbits. The same may be true for other space and laboratory plasmas. So, it is hoped that the researchers will find the moment equation description analytically more convenient to use in solving plasma physics problems where Weibel instability plays a role. But, we reiterate that the applicability of this description is limited to the hydrodynamic regime $[|\omega/(kV_{T_\parallel})|\gg 1]$, which in the context of the Weibel instability corresponds to strong temperature anisotropy.

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